

PHYS3061 Homework
Electrostatics, Partial differential equation

Question 1. *Solve the 2D Laplace equation.* Consider a rectangular domain, with two sides a and b (Figure 1). Solve the two-dimensional Laplace equation

$$\nabla^2 \phi(x, y) = 0$$

according to the following instructions:

- (1) To separate the variables, assuming $\phi(x, y) = \phi_x(x)\phi_y(y)$.
- (2) Plug back to the Laplace equation and separate it into two partial differential equations (PDE), each with only one variable (either x or y).
- (3) Set the two PDEs equal to two related constants, and solve them
- (4) The result is the “homogeneous solution”, which is independent of the boundary condition, you can then use your solution to set the boundary condition in Lab6.

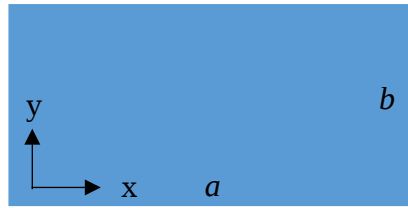


Figure 1 Schematic of the investigated domain

Question 2. *Dimensions in 2D Poisson equation.* Consider Poisson equation (or Gauss Law) in 2D

$$-\nabla^2 \phi = \frac{\rho(x, y)}{\epsilon_0}$$

the dimension (unit) of ϕ in 3D case is “voltage”. However, it is no longer the case for 2D. Assuming ρ has the dimension of “charge per area”, derive the unit of ϕ in 2D.

Question 3. *Gauss Law in 2D.* Using the above differential form of the Gauss Law and the divergence theorem (Green’s Theorem), derive the integral form

$$\oint (\vec{E} \cdot \hat{n}) dl = \frac{q}{\epsilon_0}$$

Namely, a close surface integral of the electric field will yield the charge enclosed, check and show the dimension you derived in the last problem.